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Thermalisation after inflation

Sacha Davidson and Subir Sarkar

Theoretical Physics, University of Oxford,
1 Keble Road, Oxford OX1 3NP, United Kingdom
E-Mail: davidson@thphys.ox.ac.uk, sarkar@thphys.ox.ac.uk

Abstract: During (re)heating of the Universe after inflation, the relativistic decay products χ of the inflaton must lose energy, and additional particles must be produced, to attain a thermalised state at a temperature $T_{\rm reh}$. We estimate the rate of energy loss via elastic and inelastic scattering interactions. Elastic scattering is an inefficient energy loss mechanism so inelastic processes, although higher order in the coupling α, can be faster because more energy is transfered. The timescale to produce a particle number density of $\mathcal{O}(T_{\rm reh}^3)$ is the inelastic energy loss timescale $\sim (\alpha^3 T_{\rm reh}^2/E_\chi)^{-1}$. The Universe will be thermalised within a Hubble time at $T_{\rm reh}$ if $E_\chi \lesssim \alpha^3 M_{Pl}$.

Keywords: Cosmological Phase Transitions, Physics of the Early Universe.

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Contents

1. Introduction

The primordial density fluctuations required to account for the observed large-scale structure in the universe can plausibly be generated during a quasi De Sitter expansion phase at early times [1]. The recent detection [2] of the expected characteristic signature of such fluctuations in the cosmic microwave background anisotropy provides strong evidence in favour of such an inflationary phase during which the Universe underwent superluminal expansion while dominated by the vacuum energy of a scalar field — the inflaton [3]. This period must last long enough to generate a homogeneous universe with small density fluctuations up to the scale of at least the present Hubble radius, but must eventually evolve into the hot radiation dominated era of the standard Big Bang model. The process by which the vacuum energy is converted into relativistic particles can be quite complex. Traditionally only the perturbative decay of the inflaton [4] has been considered for studying (re)heating, but there may also be non-perturbative transfer of energy to other fields — dubbed "preheating" [5]. However, this is not always an efficient energy loss mechanism for the inflaton when its decay products have self-interactions [6, 7]. Field theoretical studies of the energy transfer from a cosmological scalar field to other particles have been performed, but so far only in the context of toy models [8]. Our interest here is to determine the time-scale for the thermalisation of the bulk of the vacuum energy, when the inflaton can be taken to decay *perturbatively* into relativistic particles.

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The momentum space distribution of the relativistic decay products (χ) of the inflaton (ϕ) will not initially be thermal, so these particles must interact with each other to redistribute their momenta as well as adjust the particle number density to create a thermal distribution. This process is called "thermalisation", and has been discussed previously by many authors [15]–[21]. There are two aspects to thermalisation — reaching *kinetic* equilibrium, and achieving *chemical* equilibrium. For the first, the momentum must be redistributed among the particles present, which can happen via $2 \to 2$ scatterings and annihilations. For the second, the comoving particle number density must be modified, e.g. by decays, inverse decays, or $2 \to 3$ particle interactions. We assume here that the inflaton decay products are initially more energetic and less dense than the distribution to which they thermalise. This means that the inflaton decay products must lose energy and produce additional particles. This corresponds to generic "new" inflationary models, but possibly not to "chaotic" models where inflation occurs at field values beyond the Planck scale.

It has been noted [15, 16] that the inflaton decay products might not be thermalised at $T_{\rm reh}$, defined as the temperature when they first dominate the energy density of the Universe. Thermalisation was estimated in [15] to occur when the $\chi\bar{\chi}$ annihilation rate $\Gamma_{\rm ann}$ begins to exceed the Hubble expansion rate H, which happens well below $T_{\rm reh}$ in many inflation models. A more detailed analysis using the Boltzman equation can be found in [16]. In a numerical study of the thermalisation of a gas of semi-classical particles [17] it was found that the particles indeed reach kinetic equilibrium after a few hard scattering interactions, i.e. when $\Gamma_{\rm ann} \sim H$. However it takes longer to achieve chemical equlibrium since this requires new particle production; it was argued in [17] that the timescale for this is of order α^{-1} times the kinetic equilibration timescale.

It has been suggested that soft processes can make thermalisation faster [14, 20], because the interaction rate for processes with small momentum transfer is larger than the hard scattering rate used in ref. [15, 16]. It has also been argued that low energy particles can act as a seed for thermalising the energetic inflaton decay products, because the cross-section for annihilation with a soft particle is larger than with an energetic particle [20]. This scenario of catalysed thermalisation could be relevant if particles reach a thermal distribution via $2 \to 2$ interactions and decays. Thermalisation via annihilation and decays has been recently discussed, in a Universe where the baryon asymmetry is generated via the Affleck-Dine mechanism [21]. This is complementary to the present work, where we will concentrate on thermalisation via $2 \to 2$ and $2 \to 3$ scattering interactions.

In this paper we discuss which interactions will thermalise a bath of relativistic fermions with gauge interactions produced in inflaton decay. The thermalisation timescale is usually taken to be an interaction timescale, which begs the question "which interaction?". Thermalisation can be very fast, if it happens on the timescale of soft scattering processes, because these cross-sections diverge as the momentum transfer goes to zero. At the opposite extreme, the timescale can exceed the Hubble time, if thermalisation requires hard processes with momentum transfer of order the incident particle energy. We argue that the thermalisation timescale is at least as long as the timescale for an inflaton decay product, produced with energy of $\mathcal{O}(m_\phi)$, to lose an energy $\sim (m_\phi - T_{\rm reh})$. We estimate $\Gamma_{\rm elas}$ $(= {\rm d}(\ln E)/{\rm d}t)$, the rate of energy loss via *elastic scattering* of an energetic particle incident on other energetic particles, and find that the timescale for it to lose its incident energy is the hard annihilation timescale. We estimate the rate for energy loss due to *inelastic* $2 \to 3$ scattering, and find $\Gamma_{\rm inel} \gg \Gamma_{\rm elas}$. This suggests that $2 \to 3$ scattering interactions can thermalise the Universe faster than $2 \to 2$ processes and decays, because although higher order in α, $2 \to 3$ interactions are lower order in $T_{\rm reh}/m_\phi$. Such processes are also neccessary to bring the particles into chemical equilibrium. The timescale to produce a particle number density of $\mathcal{O}(T_{\rm reh}^3)$ via $2 \to 3$ interactions is given by $\Gamma_{\rm inel}^{-1}$, which is therefore the true thermalisation timescale.

In the next section we introduce our model and review relevant previous work [17, 20, 21]. Its purpose is to introduce notation and make this paper self-contained. In the following two sections, we discuss what interaction should be used to estimate the thermalisation rate. We study the rate of energy loss of a particle via elastic scattering in section 3. This process is solvable and only has a logarithmic infrared divergence. In section 4, we consider inelastic $2 \to 3$ processes. We estimate the timescale for an inflaton decay product to lose an energy $\sim m_\phi$, and the timescale to produce a number density $\sim T_{\rm reh}^3$. In section 5, we outline the thermalisation discussion of previous papers, which uses the annihilation rate as the thermalisation rate. We then repeat the analysis using the inelastic rate, which suggests that the Universe will indeed be thermal at $T_{\rm reh}$. We present our conclusions in section 6.

2. Model

We consider a scalar field ϕ whose potential energy is the principle component of the energy density of the Universe ρ. (We refer to ϕ as the inflaton, but it could be any other scalar field, e.g. a modulus, which dominates the universe.) The Hubble expansion rate when ϕ starts oscillating coherently about the minimum of its potential is

$$H_{\rm in}^2 \simeq \frac{8\pi}{3}\frac{\rho_\phi(a_{\rm in})}{M_{\rm Pl}^2} \simeq m_\phi^2\,. \tag{2.1}$$

The field ϕ decays at a rate $\Gamma_\phi \equiv \alpha_\phi m_\phi$ to two light fermions χ and $\bar{\chi}$.[1] The energy density of coherent scalar field oscillations redshifts like matter as the Universe expands, so the ϕ energy density ρ_ϕ then decreases with time τ as $a^{-3}{\rm e}^{-\Gamma\tau}$.

[1] Our results are largely based on dimensional analysis, so we would not expect them to change if the inflaton decay products were different.

The Universe will be dominated by the ϕ oscillations until $\tau \sim \Gamma_\phi^{-1}$, when most of the ϕ energy is transfered to the relativistic decay products.

The "reheat temperature" $T_{\rm reh}$ is defined when $H \sim \Gamma_\phi$ as

$$\rho_{\rm rad}(a_{\rm reh}) \equiv \frac{g_* \pi^2 T_{\rm reh}^4}{30}\,, \tag{2.2}$$

where g_* is the number of relativistic degrees of freedom. The χ particles are relativistic so we can define a "temperature" T for this radiation, as done above. If the χ particles have reached kinetic and chemical equilibrium, T will correspond to the thermodynamic temperature.

It is recognised [3] that there is a bath of relativistic particles prior to $T_{\rm reh}$ since ϕ decays over time and not instantaneously at $T_{\rm reh}$. There could be interesting implications for baryogenesis and other particle abundances [10]–[13] if this bath is thermalised. The number density n_χ of χ and $\bar\chi$ particles produced as ϕ decays is

$$n_\chi = 2 n_\phi(a_{\rm in})(1 - {\rm e}^{-\Gamma(\tau-\tau_{\rm in})}) \left(\frac{a_{\rm in}}{a}\right)^3 , \tag{2.3}$$

where $n_\phi(a_{\rm in}) = \rho_\phi(a_{\rm in})/m_\phi$. (This neglects χs pair-produced in χ self-interactions.) The number density of χs increases rapidly until $\tau \simeq 2\tau_{\rm in}$, then decreases as $a^{-3/2}$ until $\tau \sim \Gamma_\phi^{-1}$, when most of the inflaton energy has been transfered to the χs.

After the χs are produced, their energy redshifts. A χ produced at time τ_1 with energy $m_\phi/2$ will have energy $E_2 = (m_\phi/2)(a_1/a_2) = (m_\phi/2)(\tau_1/\tau_2)^{2/3}$ at time τ_2, so the *comoving* distribution in energy space is

$$\frac{{\rm d}N}{{\rm d}E_2} = \frac{{\rm d}N}{{\rm d}\tau_1}\frac{{\rm d}\tau_1}{{\rm d}E_2} = 6\sqrt{\frac{2E_2}{m_\phi}\frac{\Gamma\tau_2}{m_\phi}}N_\phi(\tau_1) \qquad (\tau_2 < \Gamma_\phi^{-1}), \tag{2.4}$$

where $N(\tau) = n(\tau)(a(\tau)/a_{\rm in})^3$. For $\tau_1 \lesssim \Gamma^{-1}$, we can use $N_\phi(\tau_1) \simeq N_\phi(a_{\rm in})$. The energy density in χs at some time $\tau < \Gamma_\phi^{-1}$ will be

$$\rho_\chi(a) = \int^{m_\phi/2} {\rm d}E E\frac{{\rm d}n}{{\rm d}E} \simeq \frac{3}{5}\Gamma_\phi(\tau - \tau_{in})\rho_\phi(a_{\rm in})\left(\frac{a_{\rm in}}{a}\right)^3 \equiv \frac{g_*\pi^2}{30}T^4\,. \tag{2.5}$$

The maximum χ energy density (which occurs at $\tau \simeq 2\tau_{\rm in}$) is defined to be $g_*\pi^2 T_{\rm max}^4/30$. It is easy to see that between $T_{\rm max}$ and $T_{\rm reh}$ $\rho_\chi \sim a^{-3/2}$, $T \sim a^{-3/8}$, and $(T_{\rm max}/T_{\rm reh})^4 \simeq \alpha_\phi$.

We assume that the χs have $SU(N_{\rm c})$ gauge interactions among themselves with coupling $\alpha \sim 1/30$. We would like to know how soon the χ distribution will have the equilibrium form $f(k) \sim ({\rm e}^{E/T}+1)^{-1}$. We can get a qualitative answer by comparing the expansion rate H to interaction rates, in which we make some attempt to include factors of π and $N_{\rm c}$ in sections 2 and 3. We drop them in sections 4 and 5, where the discussion is more approximate. A more accurate result could be obtained by solving Boltzman equations, or perhaps other more appropriate equations [9], for the particle phase space distributions.

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There are various $2 \to 2$ rates that could be compared to H, such as the annihilation rate, which is slow, or the scattering rate, which is fast. In the comoving rest frame, $2 \to 2$ processes redistribute energy because they do not take place in the centre-of-mass frame. They can bring a group of particles into kinetic (but not chemical) equilibrium.

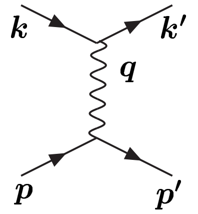

Figure 1: χ scattering. Time runs from left to right.

The annihilation cross section for χs of energy $\simeq m_\phi/2$ is

$$\sigma_{\rm ann} \simeq \frac{16 N_{\rm c} \alpha^2}{m_\phi^2}\,, \tag{2.6}$$

while the cross-section for scattering (see figure 1) is infrared divergent:

$$\sigma_{\rm scat} \simeq \frac{N_{\rm c} \pi \alpha^2}{m_\phi^2} \int \frac{{\rm d}(\sin\theta)}{(1-\cos\theta)^2} = 2\pi N_{\rm c} \alpha^2 \int \frac{{\rm d}t}{t^2}\,, \tag{2.7}$$

where θ is the scattering angle between k and k' and t is the 4-momentum transfer squared. Note that there is no thermal bath whose plasma frequency can provide a cutoff for the scattering cross-section (we are trying to compute when the thermal bath appears). In the following section, we will show that the thermalisation rate, due to scattering, is the rate $\sim n_\chi \sigma_{\rm ann}$ associated with annihilations.

The estimates in this section assume that $\alpha_\phi \lesssim m_\phi/M_{\rm Pl}$, and $m_\phi \lesssim M_{\rm GUT}$. The assumption that inflaton decay products need to lose energy and produce particles to reach a thermal distribution breaks down at $\alpha_\phi \sim m_\phi/M_{\rm Pl}$. If we approximate the equilibrium number density $n_{\rm eq} \sim T^3 \sim \rho^{3/4}$, we find, at $T_{\rm reh}$:

$$\frac{n_\chi}{n_{\rm eq}} \simeq \frac{\rho_\phi^{1/4}}{m_\phi} \simeq \sqrt{\frac{\alpha_\phi M_{\rm Pl}}{m_\phi}} \simeq \frac{T_{\rm reh}}{m_\phi}\,, \tag{2.8}$$

so $n_\chi \ll n_{\rm eq}$ if $\alpha_\phi \ll m_\phi/M_{\rm Pl}$. Perturbative particle interaction rate estimates suggest that gauge interactions are not fast enough to be in equilibrium at energy densities of $\mathcal{O}(10^{15}\,{\rm GeV})^4$, so our discussion breaks down at these energies.

3. Energy loss in elastic scattering

From the perspective of a χ particle produced in the decay of a ϕ, thermalisation is a process of losing energy. So the thermalisation timescale for χ is the timescale over which it can lose energy $\simeq m_\phi - T_{\rm reh} \sim m_\phi$ to the surrounding particles. We can estimate this timescale by integrating the particle's rate of energy loss via scattering. In this section, we consider elastic scattering. We neglect annihilations, which we do not expect to qualitatively affect our calculation, because hard scattering processes are included. Scattering interactions have been neglected in many previous thermalisation papers, who take the $2 \to 2$ annihilation rate as the thermalisation rate. We show here that the thermalisation rate due to scattering is logarithmically enhanced over the annihilation rate. So taking the $2 \to 2$ thermalisation rate $\Gamma_{\rm therm} \sim \Gamma_{\rm ann}$ is justified, but claims that scattering is irrelevant for thermalisation are not. We neglect the cosmological expansion for the following two sections to avoid the scale factor cluttering up formulae.

The elastic scattering cross-section (2.7) diverges as $\sigma \propto \mathrm{d}\theta/\theta^3$, where θ is the scattering angle. The energy exchanged is $\Delta E \simeq m_\phi \theta^2/2$, so the energy loss rate has a softer infrared divergence. The cross-section for soft glancing scattering may be very large, but the energy exchange is small, which reduces the importance of soft processes for thermalisation.

The rate of energy loss of a particle scattering on a thermal bath

$$\frac{\mathrm{d}E}{d\tau} = \langle n \sigma_{\rm scat} \Delta E \rangle, \tag{3.1}$$

where $n \sim T^3$ is an equilibrium number density and $\Delta\, E$ is the energy exchanged, has been extensively studied in finite temperature field theory [22, 23]. The logarithmically divergent result is

$$\frac{\mathrm{d}E}{d\tau} \sim 2\alpha^2 T^2 \log\left(\frac{q_{\rm max}}{q_{\rm min}}\right), \tag{3.2}$$

where $q_{\rm max}$ and $q_{\rm min}$ are the maximum and minimum momentum transfer respectively.

We follow ref. [22] in making a rough estimate of $\mathrm{d}E/\mathrm{d}\tau$ for a χ scattering on an unthermalised bath of inflaton decay products. Consider the kinematics of scattering in any frame where $\vec{p}\cdot\vec{q} \simeq 0$ (or the approximation where $\vec{p}\cdot\vec{k}' = (k_0'/k_0)\vec{p}\cdot\vec{k}$); see figure 1 for the momenta of the particles involved. We are interested in the energy lost by the relativistic χ with incident momentum k. If the 4-momentum exchanged is $q^2 = (k-k')^2 = -t$, [2] then

$$\frac{q_0}{k_0} = \begin{cases} \dfrac{2p\cdot q}{s}(1-\cos\varphi)\,, & \text{for } \vec{p}\cdot\vec{q} = 0 \\ \dfrac{2p\cdot q}{s}\,, & \text{for } \vec{p}\cdot\vec{k}' = \dfrac{k_0'}{k_0}\vec{p}\cdot\vec{k}\,, \end{cases} \tag{3.3}$$

where φ is the angle between the incident particles, and $s = 2k_0\, p_0(1-\cos\varphi)$. For p' on-shell, $q_0 \simeq k_0 t/s$. In the relativistic limit where we can neglect particle masses, eq. (3.1) can be written as [22]

$$\frac{1}{k_0}\frac{dk_0}{d\tau} \simeq \int \frac{\mathrm{d}^3 p}{(2\pi)^3} f(p)(1-\cos\varphi) \int \mathrm{d}t \frac{2\pi N_{\rm c}\alpha^2}{t^2}\frac{(k_0-k_0')}{k_0}\,, \tag{3.4}$$

where we have kept only the most infrared divergent contributions. Here $f(p)$ is the momentum space distribution of the χs, and $\mathrm{d}\sigma = 2\pi\, N_{\rm c}\alpha^2 \mathrm{d}t/t^2$ is the scattering

[2] s, t and u are the kinematical variables, τ is time.

cross-section. Using $t/s = (k_0 - k_0')/k_0$, one finds [22]

$$\Gamma_{\rm elas} = \frac{1}{k_0}\frac{{\rm d}k_0}{{\rm d}\tau} \simeq \frac{16\pi N_{\rm c}\alpha^2 n_\chi}{m_\phi^2}\ln\frac{m_\phi}{T_{\rm reh}} \sim \frac{2\pi N_{\rm c}\alpha^2\alpha_\phi^2 M_{\rm Pl}^2}{m_\phi}\ln\frac{m_\phi}{T_{\rm reh}}\,. \tag{3.5}$$

We took $q_{\rm min} \sim T_{\rm reh}$; we discuss why in the next section. The last equality is $\Gamma_{\rm elas}$ evaluated at $T_{\rm reh}$. This is logarithmically enhanced with respect to the hard annihilation rate $\Gamma_{\rm ann} \sim \alpha^2 n_\chi/m_\phi^2$.

It is easy to see in eq. (3.5) that the infrared divergence of the scattering cross-section $\sim \alpha^2 \int {\rm d}t/t^2$ is removed from the energy exchange rate because $q_0 \propto t = -q^2$ (rather than $q_0 \propto \sqrt{t}$). Elastic scattering is an inefficient energy exchange mechanism, so only speeds up thermalisation by a logarithmic factor with respect to hard annihilation processes. This agrees with the numerical results of ref. [17], who counted the number of scatterings required to bring their bath of particles to thermal equilibrium. They found that a few hard collisions per particle were sufficient, but that more soft interactions were required.

We estimate the timescale for a χ particle to lose an energy of $\mathcal{O}(m_\phi)$ by elastic scattering with other χs to be

$$\tau_{\rm elas} \sim \left(\frac{2}{m_\phi}\frac{{\rm d}k_0}{{\rm d}\tau}\right)^{-1} \simeq \left[\frac{32\pi N_{\rm c}\alpha^2}{m_\phi^2} n_\chi \ln\left(\frac{m_\phi}{T_{\rm reh}}\right)\right]^{-1}. \tag{3.6}$$

4. $2 \to 3$ scattering

There are at least two flaws in the estimate leading to eq. (3.6). Firstly, new χ particles must be created to bring the χs into chemical equilibrium. The average comoving energy per χ particle will remain m_ϕ until it can be redistributed among newly created particles. So the rate of energy exchange, which we calculated, will only be the rate of energy loss for an average χ if there are newly created χ particles available to absorb the energy. Secondly, since elastic scattering is an inefficient energy transfer process, inelastic processes, although higher order in α, could be faster.

There are various $2 \to 3$ interactions by which χs can lose energy and produce particles. We do not worry about the spin of the particles created, because this section is based on dimensional analysis. We identify particles by their energy — χs are inflaton decay products, and gs are the particles of energy $\sim T_{\rm reh}$ being created to populate the thermal bath. We refer to gs as gauge bosons, although there will be χ particles of energy $T_{\rm reh}$ in the thermal bath. We make a separation based on energy because cross-sections and number densities depend on it. We neglect gauge bosons with energy less than $T_{\rm reh}$ because they would subsequently need to be scattered up in energy to attain the equilibrium relation $\langle E\rangle \simeq 3T$. The χ particles can produce gauge bosons and lose energy via $\chi\bar\chi \to \chi\bar\chi g$, $\chi\chi \to \chi\chi\, g$ and $\chi\, g \to \chi\, g\, g$. The first process is s-channel, with $s \sim m_\phi^2$, so we concentrate on the last two t-channel processes.

There are many Feynman diagrams contributing to $\chi\chi \to \chi\chi\, g$, which can be found, with the associated matrix elements, in ref. [24]. We estimate the cross-section from the diagram of figure 1 with an outgoing gauge boson of momentum $\ell\,(\ell^2 = 0)$ emitted from the leg p'. So $p'^2 = (p'' + \ell)^2 = W^2$ is off-shell, and $p''^2 = m_\chi^2$. We neglect gamma matrices in the matrix element squared, and estimate

$$\sigma_{\chi\chi\to\chi\chi g} \sim 2\pi N_c \alpha^3 \int \frac{\mathrm{d}t}{t^2}\frac{\mathrm{d}W^2}{W^2} \sim \frac{\alpha^3}{T_{\mathrm{reh}}^2}\log\left(\frac{m_\phi^2}{T_{\mathrm{reh}}^2}\right), \tag{4.1}$$

where $(k - k')^2 = -t, 1/t^2$ corresponds to the photon propagator, and $1/W^2$ to the fermion propagator. We treat W^2 and t as independent variables, ranging from T_{reh}^2 to m_ϕ^2. We take the lower limit to be T_{reh}^2 because we are interested in making $n_g \sim T_{\mathrm{reh}}^3$ gauge bosons of energy $\sim T_{\mathrm{reh}}$. Eq. (4.1) might capture the physics of the infrared behaviour we are interested in. As $t \to 0$, it has the logarithmic divergence one expects from bremmstrahlung and a quadratic divergence due to t-channel exchange. The remainder of this paper relies on eq. (4.1) being a reasonable approximation; our arguments will not be true if $\sigma_{\chi\chi\to\chi\chi\, g}$ is suppressed by a additional factors of T_{reh}/m_ϕ due to phase space or cancellations in the matrix element.

One gauge boson for every χ can be generated via $\chi\chi$ scattering in a time

$$\tau_{\mathrm{inel}} \sim (n_\chi \sigma_{\chi\chi\to\chi\chi g})^{-1} \sim \left[\frac{\alpha^3 T_{\mathrm{reh}}^2}{m_\phi}\right]^{-1} \sim \left(\frac{T_{\mathrm{reh}}^2}{\alpha m_\phi^2}\right)\tau_{\mathrm{elas}}\,. \tag{4.2}$$

To make $n_g \sim T_{\mathrm{reh}}^3$ gauge bosons, we need to make m_ϕ/T_{reh} gauge bosons for each χ particle, as can be seen from eq. (2.8). So the timescale to produce a thermal distribution of gauge bosons via $\chi\chi$ scattering can be estimated as $(m_\phi/T_{\mathrm{reh}})\tau_{\mathrm{inel}}$. However, eq. (4.2) is also the timescale for the particles g to thermalise among themselves, at a rate $\sim \alpha^3 n_\chi/T_{\mathrm{reh}}^2$. Once $n_g > n_\chi$, gauge bosons can be produced in $\chi g \to \chi g g$ and n_g will grow rapidly, as observed in reference [17]. We can write

$$\frac{\mathrm{d}n_g}{\mathrm{d}\tau} \sim \sigma_{\chi\chi\to\chi\chi g}(n_\chi^2 + n_\chi n_g). \tag{4.3}$$

The first term will dominate until $n_g \sim n_\chi$ at τ_{inel}, then n_g will grow exponentially with a timescale τ_{inel}. So the timescale to produce a thermal number density using the cross-section (4.1) is eq. (4.2).

Cooling via the $2 \to 3$ scattering cross-section (4.1) will be faster than the $2 \to 2$ cooling rate computed in the previous section. We estimate the energy loss of the χ which emits a gauge boson to be of order $T_{\mathrm{reh}} \sim \sqrt{t}$, so

$$\frac{\mathrm{d}E}{\mathrm{d}\tau} \sim \langle (n_\chi + n_g)\sigma_{\chi\chi\to\chi\chi g} T_{\mathrm{reh}}\rangle \sim \frac{\alpha^3(n_\chi + n_g)}{T_{\mathrm{reh}}}\,. \tag{4.4}$$

The timescale to lose energy $\sim m_\phi$ through scattering on χs will be $\tau \sim [\alpha^3 n_\chi/(m_\phi T_{\mathrm{reh}})]^{-1}$. This is already a factor of $T_{\mathrm{reh}}/(\alpha m_\phi)$ shorter than the elastic cooling timescale (3.6). However, the χs can lose energy even faster by scattering off the growing bath of gauge bosons. If we substitute $n_g \sim n_\chi(a_{\rm reh}) e^{\tau/\tau_{\rm inel}}$ (approximately the solution of eq. (4.3)) into eq. (4.4), we find that the timescale for a χ to lose energy $\sim m_\phi$ is $\tau \sim \tau_{\rm inel}$. This is the timescale (4.2) to produce a thermal bath, and equals $T_{\rm reh}^2/(\alpha m_\phi^2) \times \tau_{\rm elas}$. We expect $\alpha m_\phi^2/T_{\rm reh}^2 > 1$, so $2 \to 3$ interactions cool the χs faster than elastic scattering, despite being higher order in α.

There are two disturbing features to our estimate of the thermalisation rate $\Gamma_{\rm inel} = \tau_{\rm inel}^{-1}$. It is larger than the elastic rate, which is lower order in α, and it is infra-red divergent. We can estimate the cooling timescale due to $2 \to 3$ processes of an energetic particle incident on a thermal bath. In this case, $dE/d\tau$ due to $2 \to 3$ scattering is $\sim \alpha^3 T^2$, an $\mathcal{O}(\alpha)$ correction to eq. (3.2). This is reassuring, because this thermal energy loss rate has been carefully studied [22, 23]. We find a faster thermalisation rate for our inflaton decay products at $\mathcal{O}(\alpha^3)$ than at $\mathcal{O}(\alpha^2)$ because the energy exchanged and target number densities are larger in the inelastic case. Our inelastic rate is one factor of $m_\phi/T_{\rm reh}$ larger than the elastic rate because $n_g > n_\chi$; the χ scatters more frequently off the bath of particles it created in earlier interactions. It is a second factor of $m_\phi/T_{\rm reh}$ larger because more energy is transfered in an inelastic collision than in an elastic one. We have two small parameters in our reheating problem: α and $T_{\rm reh}/m_\phi$. The inelastic thermalisation rate is higher order in α than the elastic rate, but lower order in $T_{\rm reh}/m_\phi$.

Eq. (4.4) has an infra-red divergence: the rate for cooling by emission of gauge bosons of energy μ diverges as $1/\mu^2$. Physical observables should not be infrared divergent, so a cutoff is required for our estimate. There is initially no thermal bath present to justify using the “thermal mass $\sim gT$” — but there will be at the end of (re)heating, so we imagine that the final state gauge boson must have energy of order $T_{\rm reh}$ to be on-shell. To see this, suppose that in a time $\Delta\tau$ a fraction μ^3/n_χ of the χs scatter inelastically, emitting a gauge boson of energy μ. This creates a bath of gauge bosons with number density $n_g(\mu) \sim \mu^3$, through which the next generation gauge bosons must propagate. So these next generation gauge bosons must have energy $\gtrsim \mu$. As the time interval $\Delta\tau$ lengthens, the cutoff μ grows to $T_{\rm reh}$.
We therefore require the final state gauge bosons to have energy $\gtrsim T_{\rm reh}$, and assume that the momentum transfer in the scattering is of order the energy of the emitted gauge boson. The exchanged gauge boson may not have a “thermal mass” due to interactions with other gauge bosons, because it may not live long enough to interact with them.

5. Thermalisation

We would like to know the thermalisation timescale $\tau_{\rm therm}$ after reheating. We estimate $\tau_{\rm therm} = \Gamma_{\rm therm}^{-1}$, and identify the thermalisation rate as the rate of energy loss for a χ, or the rate of particle production. (The two turn out to be comparable). In this section, we estimate when the Universe will be thermal, using the rates from the previous two sections. In the first part, we discuss two-to-to rates [15, 16, 20, 21]. We review the thermalisation mechanism suggested in reference [20], and explain where we disagree with those thermalisation estimates. We make some remarks on using decay processes as a particle production mechanism. In the second part, we compute the upper bound on the inflaton mass, below which the Universe will be thermalised at $T_{\rm reh}$ due to $2 \to 3$ scattering processes.

5.1 $\chi\chi \to \chi\chi$

The elastic rate (3.5) (without the log term , which we also drop) has been taken as the thermalisation rate in some previous work [15, 21]. This assumes that a thermal number density is rapidly produced by the decays of particles involved in the $2 \to 2$ processes. The rate scales after $T_{\rm reh}$ as $a_{\rm reh}/a$. We start in the instantaneous decay approximation to review thermalisation estimates based on eq. (3.5) [15, 20, 21]. Thermalisation will happen immediately at $T_{\rm reh}$ if

$$\alpha_\phi > \frac{m_\phi^2}{4N_{\rm c}\alpha^2 M_{\rm Pl}^2}\,. \tag{5.1}$$

For m_ϕ of $\mathcal{O}(10^3)$ GeV, the bound (5.1) translates into $\alpha_\phi > 10^{-30}$ which is certainly satisfied. However, for $m_\phi \sim M_{\rm GUT} \sim 10^{16}$ GeV, $\alpha_\phi > 10^{-4}$. For an inflaton mass of order the hidden sector scale 10^{12} GeV, eq. (5.1) implies that the inflaton decay products can annihilate with each other within a Hubble time if $\alpha_\phi > 10^{-12}$, which may not necessarily be the case [15, 21]. For instance, if ϕ decays gravitationally, $\alpha_\phi \sim m_\phi^2/M_{\rm Pl}^2 \sim 10^{-14}$. If the $\phi\chi\bar{\chi}$ interaction is of electron yukawa strength, then $\alpha_\phi \simeq 10^{-14}$.

Eq. (5.1) appears peculiar, because it gives a *lower* bound on α_ϕ. At smaller α_ϕ, the inflaton decays later, so the number density of inflatons is redshifted and therefore the number density of χs is similarly smaller ($n_\chi \simeq n_\phi$). The energy of the χs remains $m_\phi/2$, so the χ interaction rate is smaller (scales as α_ϕ^2, see eq. (3.5)). The expansion rate H is also smaller when α_ϕ is smaller, but this is a less important effect because $H \simeq \alpha_\phi m_\phi$ at $T_{\rm reh}$.

We now relax the instantaneous decay approximation and consider whether the χs produced between $T_{\rm max}$ and $T_{\rm reh}$ have time to annihilate. Repeating the estimate that lead to eq. (3.5), using n_χ from eq. (2.3) with $(1 - {\rm e}^{-\Gamma\tau}) \simeq \Gamma\tau$, and taking $E_\chi = m_\phi/2$ gives

$$\Gamma_{\rm elas}(a) \simeq \frac{32N_{\rm c}\alpha^2}{m_\phi^2}\frac{3H^2(a)M_{\rm Pl}^2}{8\pi m_\phi}\frac{2\Gamma_\phi}{3H(a)}\,. \tag{5.2}$$

If eq. (5.1) is satisfied, $\Gamma_{\rm elas} > H$ from $T_{\rm max}$ onwards; that is, if $\Gamma_{\rm elas} > H$ is true at $T_{\rm max}$ if it is true at $T_{\rm reh}$, and vice-versa. $\Gamma_{\rm elas}/H$ is a constant between $T_{\rm max}$ and $T_{\rm reh}$ because $\Gamma_{\rm elas}$ scales as $n_\chi \sim a^{-3/2}$, and H^2 scales as $\rho_\phi \sim a^{-3}$.

Now let us suppose that eq. (5.1) is *not* satisfied. As discussed in refernce [20], the annihilation rate among the first χs produced at $T_{\rm max}$ grows relative to the expansion rate H. So it is claimed [20] that these particles can interact and produce particles at some intermediate scale factor $a_{\rm max} < a < a_{\rm reh}$. More energetic particles can then thermalise rapidly, when they are produced, by interacting with this soft tail after it has reached kinetic and chemical equilibrium. This scenario is dubbed catalysed thermalisation [20]. However we disagree with some of the estimates in reference [20], because the annihilation rate of a χ of energy E_2 with less energetic χs is always smaller than the annihilation rate with more energetic χs. We can see this by evaluating those two rates at $T_{\rm reh}$:

$$\int_0^{E_2} {\rm d}E \frac{{\rm d}n}{{\rm d}E}\frac{\alpha^2}{E E_2} < \int_{E_2}^{m_\phi} {\rm d}E \frac{{\rm d}n}{{\rm d}E}\frac{\alpha^2}{E E_2} \sim 48 N_{\rm c}\frac{\alpha^2}{m_\phi E_2} n_\chi(a_{\rm reh})\,. \tag{5.3}$$

We agree that the less energetic χs are more likely to interact (the right hand side of eq. (5.3) is larger than that of eq. (3.5)), but they will annihilate with one of the energetic χs whose density is higher, hence their energy will increase. It will not substantially cool the energetic χs, because few of them interact with a less energetic χ. Nonetheless it is possible that a thermalised seed is formed out of the less energetic χs. Thermalisation is a process of energy loss and particle production, which are both assumed to occur rapidly due to decays in the annihilation and decay scenario of [20, 21]. Suppose that χ cannot decay, but can annihilate into some rapidly decaying particle ρ. An efficient cascade decay of ρ could then produce many lower energy particles that rapidly thermalise. The mass of at least one of the particles at every step of the cascade must be heavy enough that their decay rate $\Gamma_\chi = \lambda^2 m_\chi^2/E_\chi$ is larger than H. This requires some tuning of the mass, and a sufficiently large λ, but is certainly possible, particularily in supersymmetric theories where flat direction vevs could provide such an intermediate scale mass. We do not discuss further the scenario of thermalisation via annihilations and decays [20, 21]. In the next subsection, we focus on the generation of particles by $2 \to 3$ gauge scattering interactions, which should occur generically in all models.

5.2 $2 \to 3$ interactions

In section 4, we estimated the energy lost by a χ scattering and emitting a gauge boson to be $\Gamma_{\rm inel} \sim \alpha (m_\phi/T_{\rm reh})^2 \Gamma_{\rm elas}$. One factor of $(m_\phi/T_{\rm reh})$ arises because we took the inelastic energy loss in a collision to be $T_{\rm reh}$, rather than $T_{\rm reh}^2/m_\phi$ as in the elastic case. The second factor of $m_\phi/T_{\rm reh}$ comes from scattering the χs off the denser thermal bath being produced in the $2 \to 3$ interactions. We expect that $\alpha \sim 1/30$, and that thermalisation would take place within a Hubble time if $T_{\rm reh} \sim m_\phi/6$. So $\alpha\, m_\phi^2/T_{\rm reh}^2 > 1$, energy loss via $2 \to 3$ processes is faster than via $2 \to 2$ elastic scattering, and we estimate the thermalisation rate at $T_{\rm reh}$ to be $\Gamma_{\rm therm} = \Gamma_{\rm inel} = \tau_{\rm inel}^{-1}$ (see eq. 4.2). This coincides with the rate to create a number density $n_g \sim T_{\rm reh}^3$ of gauge bosons via $\chi\chi \to \chi\chi\, g$ and $\chi\, g \to \chi\, g\, g$. Thus $\Gamma_{\rm inel}$ will exceed the expansion rate at $T_{\rm reh}$ if

$$m_\phi \lesssim \alpha^3 M_{\rm Pl}\,. \tag{5.4}$$

The COBE results require the inflationary scale (and hence m_ϕ) to be much less than 10^{16} GeV [1], so this condition ought to be satisfied. As expected, this is a weaker bound than eq. (5.1). It is independent of α_ϕ because both $\Gamma_{\rm inel}(a_{\rm reh})$ and $H(a_{\rm reh})$ are proportional to α_ϕ.

Note that $2 \to 3$ interactions can thermalise the relativistic particles at $T_{\rm max}$ within a Hubble time if $\alpha_\phi > m_\phi^2/(\alpha^3 M_{\rm Pl}^2)$, which is similar to the condition (5.1).

The rate of energy loss of an energetic particle incident on a thermal bath is $\mathrm{d}E/dt \sim \alpha^2 T^2$ (see eq. 3.2) [22, 23]. The thermalisation timescale for a particle of energy m_ϕ is therefore $\sim (\alpha^2 T^2/m_\phi)^{-1}$. Our estimate of $2 \to 3$ interaction rates suggests that reheating a cold Universe is a factor of α slower; the inflaton decay products rapidly produce a bath of soft particles, and cool by interacting with them.

6. Conclusion

If the inflaton ϕ decays perturbatively in a cold Universe, its decay products must interact to thermalise. There are two aspects to thermalisation: producing additional particles and distributing energy among them, so as to obtain a kinetic and chemical equilibrium distribution in momentum space. It is often assumed that thermalisation proceeds by annihilations and decays, so the thermalisation timescale is taken to be the timescale of hard annihilations among inflaton decay products. In this paper we discussed thermalisation via scattering interactions. Our estimates suggest that these soft processes lead to faster thermalisation.

We first considered energy loss via scattering between inflaton decay products χ, and found that the timescale for a χ to cool down to $T_{\rm reh}$ is the timescale of hard annihilations. The scattering cross-section is infrared divergent so the interaction rate is large, but little energy is exchanged in soft elastic scattering so it does not lead to rapid thermalisation.

We then estimated the cooling rate via inelastic $2 \to 3$ processes $\Gamma_{\rm inel}$, and found it to be much larger: $\Gamma_{\rm inel} \sim \alpha m_\phi^2/T_{\rm reh}^2 \times$ the elastic rate. We also estimated the timescale to create a particle number density $\sim T_{\rm reh}^3$ via $2 \to 3$ interactions and found that it was of order $\Gamma_{\rm inel}^{-1}$. Thus $2 \to 3$ scattering is a more efficient thermalisation process than $2 \to 2$ elastic scattering among inflaton decay products, because the energy transfer in a collision is a factor of $m_\phi/T_{\rm reh}$ larger, and because inflaton decay products can collide with the particles they produced in earlier $2 \to 3$ interactions.

Our estimated thermalisation timescale at $T_{\rm reh}$ is

$$\tau_{\rm therm} \sim \left(\frac{\alpha^3 n_\phi}{T_{\rm reh}^2}\right)^{-1} \sim \left(\frac{\alpha^3 T_{\rm reh}^2}{m_\phi}\right)^{-1}, \tag{6.1}$$

where n_ϕ is the inflaton number density just before $T_{\rm reh}$. This result is independent of the details of the reheating model. If the Universe is dominated by relativistic particles of energy E, then they will thermalise within a Hubble time to a temperature $T_{\rm reh} \sim \rho^{1/4}$ if

$$\Gamma_{\rm therm} \sim \frac{\alpha^3}{\sqrt{\rho}} \frac{\rho}{E} > H \Rightarrow E \lesssim \alpha^3 M_{\rm Pl} \,. \tag{6.2}$$

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